# Σ In Summation

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## From the Chair

*Bob Piziak*

After seven years as Chair, Ed Oxford decided to return to the less stressful life of teaching. I was selected Interim Chair in June of 2004 for the academic year 2004-2005. I had been working as Associate Chair for a few years, so I am somewhat familiar with the work that needs to be done. Frank Mathis is now the Associate Chair and Ron Stanke is the new Graduate Director. We are all learning the ropes together.

My first task as Interim Chair was to oversee the great move of last summer. The new science building opened June 1st. We did not move to the new building, but we did take advantage of the space that opened up in Sid Richardson to bring all our faculty and graduate students under one roof. For the first time in many years, we have enough space to do the things we need to do. It took all summer, but, by the time classes began in the fall, everyone was more or less settled. We now have a reading room, a seminar room and a conference room. Everyone now has an office, no sharing necessary. We have enough classrooms to schedule classes at reasonable times for faculty and students. It is great. We have been promised that one day down the road, Sid Richardson will be refurbished and we will get the third floor. All this depends on raising sufficient funding. In the mean time, we should be fine.

Markus Hunziker joined the faculty this fall. His area is representation theory and we are very happy to have him on board. Mark Sepanski is on research leave this spring and Manfred Dugas will be on leave next spring. Mark's travels will take him to France. Three of our faculty were awarded summer sabbaticals: Dave Arnold, Paul Hagelstein and Brian Raines. John Davis began his work under his NSF grant with engineering in September. John has also successfully cleared the tenure hurdle. We have quite a few grant proposals submitted this year.

Paul Hagelstein and Ron Morgan continue their work with the Putnam Team. We had a good showing last year and hope for an even better one this year.

In January, Melvin Hood gave us a scare when he had to undergo heart by-pass surgery. Thankfully, things went well and he is recovering at home.

We were not allowed to hire any replacement faculty this year, but our search for a Chair continues. Hopefully, we will be back in the market place again next year.

A new Department of Statistical Science was created officially in June. They are located in Marrs McLean. We gave up six faculty members to this new department along with a number of courses.

Just to give a feel as to the current profile of our department, we have 18 tenured/tenure track faculty and 9 lecturers. We have 18 graduate students. Our undergraduate majors break out as 20 BA, 28 BS, 10 BS Applied, 33 BSED. The fall enrollment showed 2861 undergraduate and 54 graduate students in 88 sections of MTH 1000-4999 classes and 14 sections of MTH 5000-6999 classes. We graduated two Ph.D.'s last summer. Our faculty productivity continues to grow. We had 42 articles appear in refereed publications last year as well as much other scholarly and creative work to appear.

**In Memoriam**

Dr. Jim R. Hickey, 76, died Thursday March 24, 2005. He earned the M.S. degree in mathematics and physics from Baylor in 1959 and the Ph.D. in mathematics from the University of Texas in 1971. He joined faculty of the Baylor Mathematics Department in

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1959, attained the rank of full professor over his 33 years of service, and retired in 1992. He is survived by his wife Pat, daughter Brenda and son James. ❖

## Oxford Steps Down as Chair

*Frank Mathis*

Professor Ed Oxford did not seek a new term as chair of the Department of Mathematics and stepped down as of June 2004 to return to full time teaching. Dr. Oxford, who joined Baylor in 1982, began his service as chair in 1997. Under his leadership the department has seen tremendous growth both in size, scholarly reputation, and service to the Baylor community.

During Dr. Oxford's tenure as chair, the department appointed eleven new tenured or tenure track faculty members. These new hires have not only augmented the existing research interests of the department but also have added new areas of scholarship, in particular, analysis, differential equations and topology.

In 1998 the department added a new undergraduate degree in applied mathematics to provide students who plan to work in business and industry training that emphasizes mathematics applicable to the problems they may encounter.

The department's involvement in teacher education has increased considerably. Courses have been redesigned or added to better prepare future mathematics teachers. In particular, new courses show students how to incorporate modern technology into mathematics instruction.

The greatest change in the department has been in the graduate program. In 1997 the department offered only the master's degree and supported between five and eight graduate students each a year. In 2000 the department was approved to offer the Ph.D. degree. Since then the department has graduated three Ph.D. students and the graduate population has grown to 18 students of whom 14 are working toward a doctorate.

The department is grateful for the strong leadership that Dr. Oxford has provided during these past seven years. Dr. Bob Piziak is currently serving as interim chair while the department searches for a replacement. ❖

> A mathematician is a device for turning coffee into theorems.
>
> - Paul Erdös -

## Erdös Numbers

*Manfred Dugas*

While Paul Erdös was a famous mathematician, Erdös numbers are **not** a particular kind of number named after him because he discovered and/or studied them. No, things are more interesting than that. First let me tell you some things about the mathematician Paul Erdös. He was born March 26, 1913, in Budapest, Hungary. His parents were Jewish, but did not observe the Jewish religion. He had two older sisters, but both died of scarlet fever just a few days before he was born. That made his parents extremely protective of Paul and a German governess was hired, so he could be home schooled. Both his parents were mathematics teachers and they introduced Paul to this subject at an early age.

The political situation in Hungary was quite chaotic after World War I and life was hard, especially for a Jewish person. In 1920, anti-Jewish laws were introduced similar to the ones of Nazi Germany thirteen years later. Despite the restrictions placed on Jews, Erdös was allowed to attend the Pázmány Péter University in Budapest, where he earned a doctorate in mathematics in 1934.

He was forced to leave Hungary and took a post-doctoral position at Manchester, England. In 1938, it became clear that he wouldn't be able to return safely to Hungary, and he accepted a fellowship at Princeton. Princeton found him "…uncouth and unconventional…" and did not renew his fellowship. The famous mathematician Ulam invited Erdös to Madison providing him with a roof over his head. He was extremely productive and posed and solved many problems in number theory and other areas of mathematics. Usually, he posed and solved problems that were beautiful, easy to understand, but notoriously difficult to solve.

In August 1941, Erdös went for a walk with two fellow mathematicians and, deep in thought, did not notice a NO TRESPASSING sign. They got too close to a military radio transmitter and were picked up by police. The innocent nature of the incident was determined quickly, but it got him an FBI record.

In 1943, Ulam left Madison to work on the Manhattan Project. He asked Erdös to join him, but Erdös gave the 'wrong answers' at his interview and was rejected. It is not clear if Erdös was too honest, or if he just had a bit of fun with the interviewers. That year, he took a part time job at Purdue. In 1951 he won the American Mathematical Society's *Cole Prize.*

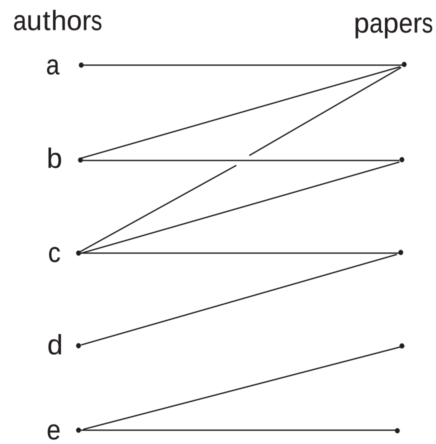
After Hungary was liberated, he learned that his family had suffered terribly under Nazi rule, and he was able to return to Hungary for a visit in 1948. He accepted a temporary post at Notre Dame in 1952, which gave him complete freedom to do research.

In the early 1950s he became entangled in Senator McCarthy's communist witch hunt. Returning from a conference in Amsterdam in 1954, he gave a 'wrong answer' to the US immigration's question regarding what he thought about Karl Marx: "I am not competent to judge, but no doubt he was a great man". Erdös, without any reason given, was not allowed back into the United States.

He spent much of the next ten years in Israel and, after several requests, finally received a visa in 1963. At that time he had become a mathematical vagabond, moving from one university to another, and from one friend's home to the next. He won the $50,000 Wolf prize in 1983, but had a lifestyle that needed little money, and he donated most of his money to help students. He died on September 20, 1996, of a heart attack while visiting Warsaw, Poland.

Paul Erdös (co)authored about 1,500 papers with about 500 coauthors. This makes him the most productive mathematician of all time. This information is based on the records in the American Mathematical Society's Math Reviews database, which contains about 1.9 million authored items by a total of about 401,000 authors.

Now we get to our peculiar topic, considered fun by mathematicians, who find many things interesting that often don't appeal to the uninitiated. Let ***B*** be the (bipartite) graph whose **vertices** are papers and authors (as found in the Math Reviews) and there is an edge connecting an author with a paper if and only if the author is the (a) (co)author of the paper. Then ***B*** has about 2.9 million edges!! Given two authors X and Y, then X and Y are **connected** (in ***B***) if there is a path of edges going from X to Y. The **distance** between X and Y is one half of the least number of edges of any path between X and Y. The following is a hypothetical subgraph of ***B***.




| author | distance to (a) |
|---|---|
| a | 0 |
| b | 1 |
| c | 1 |
| d | 2 |
| e | ∞ |

One giant connected component of ***B*** is ***E***, the one containing Paul Erdös. As a consequence, mathematicians became interested in their **Erdös Number**, which is their distance to Paul Erdös in ***B***. For example, if you are Paul Erdös, then you Erdös number is 0; if you have a joint paper with Paul Erdös, then your Erdös number is 1; if you have a joint paper with a coauthor whose Erdös number is 1, and you don't have a joint paper with Paul Erdös, then your Erdös number is 2; etc. In general, if you have a joint paper with a coauthor whose Erdös number is n, but not with a coauthor whose Erdös number is less than n, then your Erdös number is n+1. Authors not in ***E*** have Erdös number ∞.

In case you are wondering, here is a list of Baylor's mathematics faculty with a finite Erdös number.

Erdös number 2: M. Dugas.

Erdös number 3: D. Arnold, P. Hagelstein, J. Henderson, E. Oxford, M. Sepanski.

Erdös number 4: J. Davis, M. Hunziker, B. Johns, R. Morgan, B. Piziak, R. Stanke, K. Kirsten.

Erdös number 6: F. Mathis.

It is not very clear what Erdös' view on religion was. He might have placed himself somewhere close to the triangle of atheism, deism, and agnosticism. He seemed to have believed in some afterlife. He is quoted as saying: "…Maybe, once I've left, I'll be able to be at many places at the same time. Maybe then I'll be able to collaborate with Archimedes and Euclid." He frequently referred to God as holding "The Book" containing all mathematical knowledge. When he saw an especially elegant proof, he often said: "This is one from The Book."

To finish this article, let me mention two more quotations attributed to Erdös.
"Why are numbers beautiful? It's like asking 'why is Beethoven's Ninth Symphony beautiful?' If you don't see why, someone can't tell you. I *know* numbers are beautiful. If they aren't beautiful, nothing is."
"Television is something the Russians invented to destroy American education."

**Acknowledgements**
The information in the article was obtained from the following websites.

www.history.mcs.st-and.ac.uk/history/Mathematicians/Erdos.html
(This is a short biography of Paul Erdös)

www.oakland.edu/enp/trivia.html.
(Here you find all you ever want to know about ***B***.)

www.ams.org/msnmain/cgd/index.html
(Here you can find the distance between two authors in ***B***.)

**Paul Erdös was an extreme mathematician, 24/7!** ❖

## Graduate Student Profile

*Michael Coons*

A cattle ranch in the mountains of northwestern Montana may not be the likeliest location to hatch a mathematician, but Double K Land and Cattle is an exceptional place. I came of age there, moving irrigation pipe in circles, picking sticks and stones from the pastures and reading what my father deemed to be the classics, one a week, until I graduated with honors from Flathead High School. In those days I learned the value of hard work, the pleasure of books, and that I'd rather make a living with my mind than with my calloused oil-stained hands.

I cannot remember a time when I wasn't fascinated by ordered systems in language, music, mathematics, or anywhere else they cropped up in the world around me. From early explorations of the rivers and highways of the western states, to forays into chemistry, French and German classes in college, I've been looking for connections, for explanations.

The greatest satisfaction I have derived in this search has come from mathematics. The thrill of gliding through a well-executed proof rivals the adrenaline rush of careening down Going-to-the-Sun road in Glacier National Park on a moonlight bike ride. I just can't get enough of it.

I've also come to enjoy navigating other cultures and geographies. Two-and-a-half years ago I married my wife, Alissa, who spent her formative years overseas. Not only has she lured me to study in Europe (for the first time in Switzerland in 2002), we've also formed our own small system, two as one, to relay through the layers of education and career.

We graduated from the University of Montana in 2003, and came to Waco in January 2004 so I could pursue my Master's degree at Baylor. While it was difficult to leave the mountains, I have been very pleased with the quality of the department and the mathematics I am experiencing here.

My first semester at Baylor was taken up by core requirements. In the Summer I took a reading course of Analytic Number Theory with Dr. Klaus Kirsten, thinking I might pursue something in this area as a thesis topic. Within this reading, we focused mainly on Zeta-functions, which led nicely into the study of partitions of integers. Due to Dr. Kirsten's extensive experience in this area (see 2004 edition of In Summation), we came up with some really beautiful results, called the General Moment Theorems. These theorems give the moments of partitions over nondecreasing sequences of integers. They are the basis of my Master's thesis, *General Moment Theorems with Applications*. With the completion of my thesis, I am slated to graduate in May 2005.

I came to Baylor hoping to get good research training, I have received much more than I could have ever dreamed. The faculty and staff of the Baylor mathematics department have been extremely helpful to me and my peers. The openness to students and the commitment to research of professors such as Dr. Kirsten have inspired and motivated me to further studies in mathematics. I hope to be able to provide such guidance to students of my own some day.

As for my future plans, I have been awarded a 2005-2006 Fulbright fellowship, where I plan to continue my expedition into Number Theory at the Alfréd Rényi Institute of the Hungarian Academy of Sciences. After this year abroad, I will continue to pursue a Ph.D. at an institution yet to be determined. After obtaining my Ph.D., I would like to continue doing research in mathematics, perhaps at a university somewhere in the Pacific Northwest. ❖

## Scholarships

*Frank Mathis*

The Department of Mathematics awarded $61,000 in scholarships to twenty undergraduate mathematics and mathematics education majors for the 2004-2005 academic year. Money for the scholarships came from several endowed funds that are supported by friends and alumni of the department. These include the Earl, Maxine, Max, and Anita Bodine Fund, the K. L. and Vivian Carter Fund, the Jerry Johnson Scholarship Fund, the Howard L. Rolf Fund, the Hickey- Piziak Scholarship Fund, and the Ruth and Gene Royer Mathematics Scholarship Fund.

Students receiving awards are Brittney Benefield, Teresa Bland, Sarah Colquitt, Rachel Grubb, Amy Johnson, Lindsay Longsine, Jeff Lyons, David McCune, Lauren Mitchell, Damilola Olupona, Courtney Owen, Clint Prater, Amanda Schwada, John Scott, Stephanie Shaw, Casey Sherman, Julie Solomon, Michelle Verner, Ashley West, Alise Williamson.

The department usually solicits applications each March for the following year's scholarships. Students may pick up an application form in the Mathematics office (SR 338). For more information concerning scholarships, students may contact Dr. Frank Mathis at 710-6569. ❖

## The Wobbly Table Problem

The Wobbly Table Problem

*Markus Hunziker*

If you listen to mathematicians you will probably notice that they frequently use words such as “beautiful” or “amazing” when they are talking about mathematics. “What is beautiful about mathematics?” you might ask. Usually a mathematician exclaims that a theorem or a proof is beautiful after a moment of sudden understanding or “seeing the light.” Unfortunately, it is quite difficult for mathematicians to share this kind of experience with non-mathematicians. In the following, I will try anyway.

Here is an example of a very practical problem with an absolutely beautiful solution that I learned as a student from Hanspeter Kraft at the University of Basel (see [K]).

> What is it indeed that gives us the feeling of elegance in a solution, in a demonstration? It is the harmony of the diverse parts, their symmetry, their happy balance; in a word it is all that introduces order, all that gives unity, that permits us to see clearly and to comprehend at once both the ensemble and the details.
> - Henri Poincaré -

**The problem.** *It is a nice day and you decide to have a cup of coffee or a small meal outside at a street cafe. Upon sitting down at the table you notice that the table is wobbling. What do you do?*

The common solution to the problem is to find some napkins or cardboard and to fill the gap between one of the legs of the table and the ground. Usually this works reasonably well, but I think we agree that this solution is not particularly beautiful. So let me present to you the mathematician's solution.

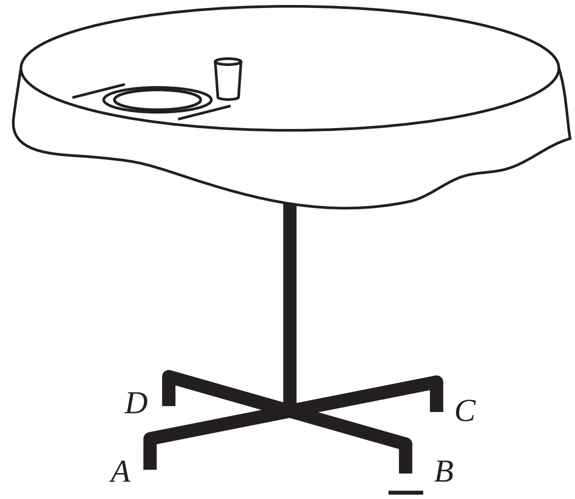
**The mathematician's solution.** *Assuming that the four feet of the table form a square, you simply have to start rotating the table (about a vertical axis) and before you have completed a 90 degree rotation the table will have stopped wobbling!*

Surprised? You may of course not believe it at first. Perhaps you can think of an instance were the claim seems false. For example, suppose the floor is perfectly even but the table is not fair in the sense that one leg is too short (or too long). In this case there is indeed not much you can do and you may have to start looking for napkins. In practice, however, the table is usually fair and it is simply wobbling because it is placed on uneven ground. This was in fact implicit in our assumption that the four feet of the table form a square since by definition the vertices of a square lie in a plane. We also have to be a little bit careful about how uneven the ground is. We will assume that there are no discontinuities such as sharp steps, etc. Again, this is a very reasonable assumption in practice.

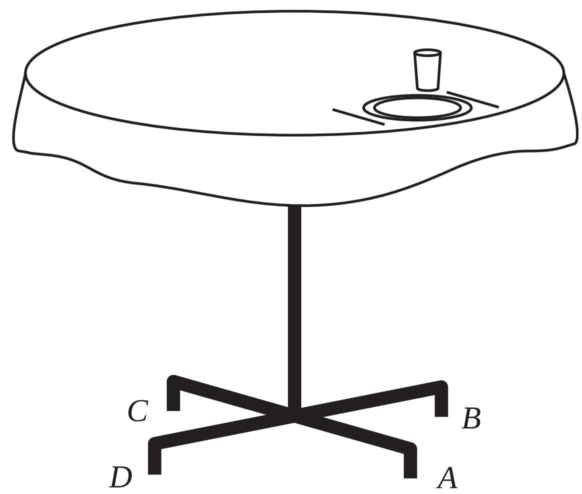
So now that we agree on our assumptions let us start thinking about why it might work. First we need to understand what wobbling means. Clearly the table will be steady precisely when all four legs are on the ground. Thus, the table will wobble if there is at least one leg off the ground. If a leg is off the ground it can be pushed to the ground, but then another leg is moved off the ground. The table behaves like a seesaw. But *about what* is the table wobbling (*i.e.*, pivoting)? After a little bit of thinking, we will realize that there is a pair of opposite legs that always stays on the ground, while the other two legs can be moved up and down. So the table wobbles about the axis given by the line through the two feet that stay on the ground. Note that there are *two possible axes* about which the table can wobble, corresponding to the two diagonals of the square. Let us summarize our observations as follows. If the feet of the table are labeled $A$, $B$, $C$ and $D$ (say counterclockwise), then there are three possible states the table can be in:

(1) The table will not wobble,
(2) The table will wobble about axis $AC$, or
(3) The table will wobble about axis $BD$.

Now suppose our table wobbles initially about axis $AC$, *i.e.*, legs $A$ and $C$ are on the ground, while leg $B$ (or leg $D$) is off the ground.

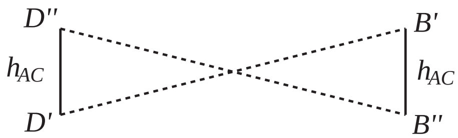



What happens if we rotate the table 90 degrees, say counterclockwise? Well, after the rotation, foot $A$ is where foot $B$ was, foot $B$ is where foot $C$ was, etc.




Aha! This means that the table is now no longer wobbling about axis $AC$, but about axis $BD$. Can we then conclude that, at some intermediate position, the table was steady? As we will see shortly, we can by what is called a *continuity argument*.

To set up a continuity argument we need to introduce a measure of *how much* the table is wobbling in a given state. If the table is wobbling about axis $AC$ then foot $B$ (and foot $D$) can be moved up and down between two extreme positions, say $B'$ and $B''$ (and $D'$ and $D''$, respectively). If the table does not wobble about axis $AC$ we can still define $B'$ and $B''$ (and $D'$ and $D$): in this case $B' = B''$ (and $D' = D''$). In either case, let us define the *wobbling amplitude* $h_{AC}$ as the distance between the two points $B'$ and $B''$ (which is the same as the distance between $D'$ and $D''$).




Similarly, we can define the wobbling amplitude $h_{BD}$, which measures how much the table is wobbling about axis $BD$.

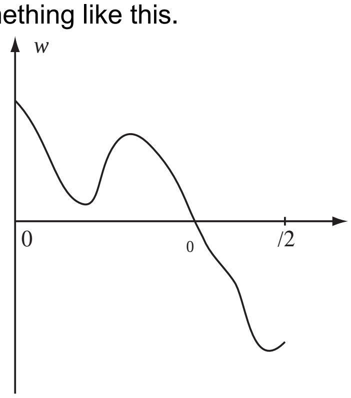
Recall from above that there are three possible states the table can be in: (1) the table will not wobble, (2) the table will wobble about axis $AC$, and (3) the table will wobble about axis $BD$. In terms of wobbling amplitudes these possible states correspond to the following.

(1) $h_{AC} = 0$ and $h_{BD} = 0$
(2) $h_{AC} > 0$ and $h_{BD} = 0$
(3) $h_{AC} = 0$ and $h_{BD} > 0$

This can be further simplified if we consider the difference $w = h_{AC} - h_{BD}$. Then the three possible states are as follows.

(1) $w = 0$
(2) $w > 0$
(3) $w < 0$

Let us now look at the values of $w$ as we rotate the table starting from an initial position where the table is wobbling about axis $AC$. If $\theta$ denotes the angle of rotation, let $w(\theta)$ denote the corresponding value of $w$. Since the table is initially wobbling about axis $AC$, we have $w(0)>0$. Since after a 90 degree rotation, the table is wobbling about axis $BD$, we have $w(\pi/2)<0$. Here is the key observation. By our assumption that the ground does not have any sharp steps etc., the value of $w$ varies *continuously* with $\theta$. You may take a breath and think about this for a moment: if we change the position of the table just a tiny bit, the wobbling amplitudes and, hence, $w$ will also only change a tiny bit and will not abruptly jump. This is what we mean by saying "varies continuously". Thus the graph of $w$ as a function of $\theta$ is a continuous (i.e. uninterrupted) curve looking something like this.




Since $w(0)>0$ and $w(\pi/2)<0$, there must be an angle $\theta_0$ between 0 and $\pi/2$ such that $w(\theta_0) = 0$! So, if we rotate the table and reach the angle $\theta_0$, the table is not wobbling. This proves that the method works. Try it the next time you sit at a wobbly table!

Let me close by briefly mentioning a problem related to the one above. You may have noticed that we didn't require that the top of the table be horizontal when it is in a non-wobbling position. In practice, the ground is usually not too far off from being a horizontal plane, and hence the table will be almost horizontal. But suppose you have a floor that looks like a serious hill. Can you move the table into a position on the hill such that all four feet touch the ground and the top of the table is horizontal? (The legs of the table should be long enough if you want to try this.) There is a beautiful theorem due to Roger Fenn (see [F]) that says that you can. It assumes, however, that the hill is located within some bounded region, say a disk, and that the floor is perfectly flat and horizontal outside the disk. Here is the mathematically precise version of the theorem.

**Theorem** (Fenn's Table Theorem) *Let $f$ be a non-negative continuous function on* $\mathbf{R}^2$ *such that* $f = 0$ *outside a disk* $D$ *and let* $s$ *be a positive real number*. *Then there are four points in* $\mathbf{R}^2$ *forming the vertices of a square with center in* $D$ *and with sides of length* $s$, *such that* $f$ *takes the same value at the vertices of the square*.

The proof of this result is more difficult than the proof we gave above, but anyone who has taken an introductory course in algebraic topology should be able to understand and enjoy it. If you haven't taken such a course, well, perhaps you should: the view is breathtaking.

**Exercises**

Here are two exercises for you to practice the use of continuity arguments.

1. You and your friend order a pepperoni pizza. When the pizza arrives you notice that the pepperoni is very unevenly distributed. Show that you can still cut the pizza along a diameter such that both halves are covered by exactly the same amount of pepperoni. (The knife is sharp enough to slice through a piece of pepperoni.)

2. Show that at any given instant there is pair of antipodal (i.e., diametrically opposed) points on the surface of the globe at which the temperature is exactly the same.

**References**

[K] Hanspeter Kraft, *The wobbly garden table*, J. Biol. Phys. Chem. 1 (2001), 95-96
[F] Roger Fenn, *The table theorem*, Bull. London Math. Soc. 2 (1970), 73-76 ❖

# Funding Opportunities

*Bob Piziak*

Budgets at Baylor continue to be tight. We appreciate very much the financial support people have provided us over this past year. Many opportunities, large and small, exist to help us grow towards our vision. Please contact the department at 254-710-3561 if you would like to discuss a way you might be able to help us. In the mean time, we want to say a great big THANK YOU to those who made financial contributions to the department last year. These include

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In honoring the memory of Dr. Jim Hickey, his family requests that donations be made to the endowed scholarship fund established by Jim and Pat in mathematics, the women's basketball program at Baylor or the Foundation Fund of First Baptist Church Waco. ❖

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